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ELEMENTS IN REGION OF TUNGSTEN

FORMED BY FUSION IN A FISSION BOMB

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Principles of Testing

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The fission products in a bomb at the time of the last few generations before explosion are sufficiently numerous that the probability of reaction with each other becomes measurable. In this paper the probability of fusion will be calculated. It is found that the fusion of two light fragments is the only possible process of this type. For a bomb containing 14 kg of oralloy or plutonium and having a 25% efficiency the yield of platinum group elements is estimated as 1.2×10^{14} atoms. Although the nucleus produced instantaneously with highest probability is 75-190, the excess of kinetic energy is such as to boil off between 9 and 18 neutrons forming elements 75-175 to 75-182. Therefore the nuclei formed will be on the low side of the stability curve. Most nuclei in this region decay by K-capture although two positron emitters, ${}_{71}Lu^{170}$ and ${}_{69}Tm^{166}$, are known and perhaps others may be discovered.

Consider an average light fragment, Z = 37, A = 96, and an average heavy fragment, Z = 55, A = 138, which have just separated from each other. The average kinetic energy of the light fragment is $\frac{138}{234}(168) = 99$ mev, and of the heavy fragment is $168 - 99 = 69$ mev.

A freshly formed light and heavy fragment have almost enough energy to recombine to form the original nucleus 92-235. But two heavy fragments are unable to combine to form a transuranic element, e.g. 110-276, for the reason that the reaction is endothermic by more than the maximum relative kinetic energy of the two, e.g. 138 mev.

The situation is more favorable for the light fragments. Two freshly formed light elements which collide may react to form such an element as 74-192. The mass difference between nucleus 74-192 and two

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nuclei 37-96, calculated from the Metropolis mass table, has the value 89 mev. The height of the coulomb barrier* is $0.96 \left\{ \frac{(37)^2}{2(96)^{1/3}} \right\} = 143$ mev. The initial relative kinetic energy for a head on collision of two average light fragments is 2 x 99 = 198 mev, which is greater than both the mass difference and the barrier. The barrier height, being greater than the mass difference is taken here as the limiting factor; we assume that a light fragment may react until it has lost of the order of $\frac{1}{2}(198 - 143) = 27.5$ mev, i.e. until it has energy 71.5 mev.

Estimate of the probability of fusion of two light fragments

In order to take into account the collisions of two fragments of unequal energy in the energy range in which fusion may occur, we define a quantity q as the total number of light fission fragments per cubic centimeter and per unit volume of velocity space having velocity between v and v + dv.

1) Calculation of q: Let N be the total number of light fission fragments produced during the duration T of fission in the bomb (T ~ 2 shakes). For 25% yield in a bomb containing 14 kg of Pu or oralloy,

$$N = \frac{1.4 \times 10^4}{235} \times \frac{1}{4} \times 6 \times 10^{23} = 8.91 \times 10^{24}$$

Let Δt be the time for a light fission fragment to lose 1 cgs unit of velocity. Then at any instant, n(v)dv, the total number of light fission fragments with velocity between v and v + dv, is $N \Delta t / T$.

To estimate Δt, the rate of energy loss of a light fission fragment to a 10 kev Maxwellian distribution of electrons is needed; this may be calculated from LA-401. Assume the reaction to occur mainly at normal

*Computed on the assumption that the nuclear radius is $1.5 \times 10^{-13} A^{1/3}$ cm.

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density.

$$-\frac{dE}{dx} \cong N_1 \frac{4\sqrt{\pi} Z_1^2 e^4}{\sqrt{ET}} \sqrt{\frac{1}{1830\ A}} \left\{\frac{2}{3}\frac{E}{T}\right\} \ln \frac{P_1}{P_0}$$

$$N_1 = 92 \times \frac{19}{235} \times 6 \times 10^{23} = 4.46 \times 10^{24} = \text{density of electrons}$$

$$\frac{P_1}{P_0} = \frac{m^{3/2}\, v^2_{rel}}{h\sqrt{4\pi N_1 e^2}} = \frac{\left(9 \times 10^{-28}\right)^{3/2} \left(2.8 \times 10^9\right)^2}{1.06 \times 10^{-27} \sqrt{4\pi 4.46 \times 10^{24}\ 23 \times 10^{-20}}} = 56$$

$$\ln \frac{P_1}{P_0} = 4.02$$

$$E = 99 \text{ mev},\ T = 10 \text{ kev},\ A = 96,\ Z = 37$$

$$-\frac{dE}{dx} = \frac{2}{3} \frac{4.46 \times 10^{24}\ 4\sqrt{\pi}\,(37)^2 \left(4.8 \times 10^{-10}\right)^4 \left(99 \times 1.6 \times 10^{-6}\right)^{1/2}}{\left(10 \times 1.6 \times 10^{-9}\right)^{3/2} \quad (1830 \times 96)^{1/2}} \times 4.02$$

$$-\frac{dE}{dx} = .0926 \text{ ergs/cm}$$

The time to lose 1 cgs unit of velocity is

$$\Delta t = \frac{1}{\left(\frac{dv}{dt}\right)} = \frac{M}{\left(\frac{dE}{dx}\right)} = \frac{98 \times 1.66 \times 10^{-24}}{.0926} = 1.76 \times 10^{-21} \text{ seconds}$$

$$n(v)dv = N \frac{\Delta t}{T} = 8.91 \times 10^{24}\ \frac{1.76 \times 10^{-21}}{2 \times 10^{-8}} = 7.85 \times 10^{11}$$

The quantity q is defined per unit volume of velocity space and per cm^3 and so contains the reaction volume $V = \frac{1.4 \times 10^4}{19} = 736\ cm^3$.

$$q = \frac{n(v)}{4\pi v^2}\ \frac{1}{V} = \frac{7.85 \times 10^{11}}{4\pi\ \left(1.4 \times 10^9\right)^2}\ \frac{1}{736} = 4.31 \times 10^{-11}$$

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2) Total velocity space from which collisions effecting fusion can occur against a given element of velocity space: For an average light fission fragment the maximum velocity is

$$v_0 = \sqrt{\frac{2 \times 99 \times 1.6 \times 10^{-6}}{96 \times 1.66 \times 10^{-24}}} = 1.41 \times 10^{9} \text{ cm/sec}$$

The minimum relative velocity at which a fusion can occur is

$$V_1 = \sqrt{\frac{2 \times 143 \times 1.6 \times 10^{-6}}{\frac{96}{2} \times 1.66 \times 10^{-24}}} = 2.39 \times 10^{9} \text{ cm/sec}$$

Consider a fission fragment of velocity v in a particular increment of velocity space d^3v. It may produce a fusion by collision with any particle lying in the velocity space inside the sphere of radius v_0 about the origin and outside the sphere of radius V_1 circumscribed about d^3v. This volume between the two spheres is given by

$$\int_0^{\sqrt{2\left(v_0 + v - V_1\right)/\left(\frac{1}{v_0} - \frac{1}{V_1}\right)}} \left\{ \left(v_0 + v - V_1\right) + \frac{x^2}{2V_1} - \frac{x^2}{2v_0} \right\} 2\pi x \, dx$$

$$= \pi \left\{ \frac{\left(v_0 + v - V_1\right)^2}{\frac{1}{v_0} - \frac{1}{V_1}} \right\}$$

3) Total number of collisions:

A simplifying approximation follows from the fact that $V_1 \sim 2v_0$. Since the upper limit on the velocity v of a fission fragment is v_0, the lower limit is $V_1 - v_0$ which is not greatly different from v_0. On this account, in the integral below, v may be replaced by $\frac{V_1}{2}$, and V_1 may be

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substituted for v_{rel}:

$$\text{Collisions per second per cm}^3 = \frac{1}{2}\int_{V_1 - v_0}^{v_0} \sigma v_{rel}\, q^2 \left\{ \frac{\pi\left(v_0 + v - V_1\right)^2}{\frac{1}{v_0} - \frac{1}{V_1}} \right\} 4\pi v^2 dv$$

$$\sim \frac{1}{2}\sigma V_1 q^2\, 4\pi^2 \left(\frac{V_1}{2}\right)^2 \frac{V_1 v_0}{V_1 - v_0} \int_{V_1 - v_0}^{v_0} \left(v_0 + v - V_1\right)^2 d\left(v_0 + v - V_1\right)$$

$$\sim 2\pi^2 q^2 \sigma \frac{V_1^{\,4}}{4} \frac{\left(2v_0 - V_1\right)^3}{3}$$

In this expression σ is the cross section for fusion of two light fission fragments. It is not known, but is assumed to be one barn for two fragments having relative energy greater than the Coulomb barrier.

The total number of collisions is

$$TV\left\{ 2\pi^2 q^2 \sigma \frac{V_1^{\,4}}{4} \frac{\left(2v_0 - V_1\right)^3}{3} \right\}$$

$$\left(2v_0 - V_1\right) = 0.43 \times 10^9 \text{ cm/sec}$$

Total number of fusions =

$$(736)\ 2 \times 10^{-8} \left\{ 2\pi^2 \left(4.31 \times 10^{-11}\right)^2 10^{-24} \frac{\left(2.39 \times 10^9\right)^4}{4} \frac{\left(.43 \times 10^9\right)^3}{3} \right\}$$

$$= 1.2 \times 10^{14}$$

<u>Distribution of fusion nuclei</u>

This number of atoms is distributed among several nuclear species. In order to get an idea of this distribution we may fit the light fission fragment yield curve roughly by a Gaussian (where A is the mass number of

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a chain):

$$\exp\left\{-\left(\frac{96 - A_1}{8}\right)^2\right.$$

The probability of formation of element $A = A_1 + A_2$ is proportional to

$$\int \exp -\left(\frac{96 - A_1}{8}\right)^2 \exp -\left(\frac{96 - A_2}{8}\right)^2 d\,A_1 \cong \exp -\left(\frac{192 - A}{11.3}\right)^2$$

except for the effect of variation in the Gamow barrier, and of increase in initial kinetic energy of a fission fragment as its mass decreases. These two factors affect the solid angle over which collisions may occur to form a fusion.

In general most collisions are not head on. The relative kinetic energy of a collision of two particles of equal energy depends on the angle at which they collide. The relative velocity must be averaged over all angles for which E_{rel} is greater than the Coulomb barrier. This correction may be made crudely as follows. Let the angle between the two colliding fragments in the lab system be θ, and assume for simplicity that the two fragments have equal energy E. Then:

$$v_{rel} = 2v \sin \theta/2$$

$$E_{rel} = \frac{1}{2}\frac{M}{2} v^2_{rel} = 2\left\{\frac{1}{2} M v^2\right\} \sin^2 \frac{\theta}{2}$$

$$E_{rel} = 2E \sin^2 \frac{\theta}{2}$$

For each value of E the limiting angle θ such that E_{rel} is not less than E_B, the height of the barrier, is given by

$$\sin \theta/2 = \sqrt{\frac{E_B}{2E}}$$

The average value of the relative velocity times the effective solid angle:

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$$(\Delta\Omega)v_{rel} = 2v \int_{\theta}^{\pi} \frac{1}{2} \sin\frac{x}{2} \sin x \, dx$$

$$= \frac{4}{3}\, v \left(1 - \sin^3 \frac{\theta}{2}\right) = \frac{4}{3}\, v \left\{1 - \left(\frac{143}{2E}\right)^{3/2}\right\}$$

For example consider the element 82-208 to be made by fusion of two atoms 41-104. The barrier is

$$.96 \left\{ \frac{(41)^2}{2(104)^{1/3}} \right\} = 171 \text{ mev}$$

The kinetic energy of each fragment is $\frac{235 - 104}{235} \times 168 = 94$ mev.

The maximum relative kinetic energy of the two fragments is 188 mev. Therefore fusion can occur until each fragment has lost $\frac{1}{2}(188 - 171) = 8.5$ mev. The effective mean energy of the fragment is

$94 - \frac{8.5}{2} = 90$ mev. The effective

$$(\Delta\Omega)v_{rel} = .093 \, v$$

For the formation of 76-192 from two atoms of 37-96 we have already found the barrier to be 143 mev and the mean energy 85.3 mev. One obtains

$$(\Delta\Omega)v_{rel} = .31 \, v$$

At the light end of the spectrum we now need the correction for formation of 70-176 from two fragments 35-88. The Gamow barrier is

$$.96 \left\{ \frac{(35)^2}{2(88)^{1/3}} \right\} = 133 \text{ mev}$$

The initial kinetic energy is

$$\frac{235 - 88}{235} \times 168 = 105 \text{ mev.}$$

The maximum kinetic energy is 210 mev, therefore each fragment can lose

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no more than $\frac{1}{2}(210 - 133) = 38.5$ mev. The effective energy of each fragment is $105 - \frac{1}{2}\,38.5 = 86$ mev. The effective correction factor is

$$\frac{4}{3}\left\{1 - \left(\frac{133}{2 \times 86}\right)^{3/2}\right\} = .43$$

$$(\Delta\Lambda)v_{rel} = .43\ v$$

A probable distribution of fusion products is shown below. The relative probability of formation has been computed for the instantaneous A value. However for all these nuclei there is enough kinetic energy in excess of the energy needed for the reaction that many neutrons will boil off.

The average binding energy of a neutron in this region is 6 mev. At the heavy end, e.g. Au, the available KE ranges from 171 to 188 mev, and the endothermic energy of fusion is 101 mev. Therefore between 12 and 14 neutrons will boil off the nucleus 79-202 producing nuclei Z = 79 and A = 190 - 188. Since these elements are far below the curve of stability they will decay rapidly by K-capture or by positron-emission. The significant quantity will then be not Z but A.

Similarly, in the region of the most probable instantaneous element 75-190, the available KE is 198 to 143 mev and the mass difference is 89 mev. There will be from 9 to 18 neutrons boiled off producing mass chains of 172 to 181. And at the light end of the distribution, e.g. 70-178, the available KE ranges from 210 to 133 mev, while the mass difference is 80 mev. One expects therefore masses ranging from 169 to 156 corresponding to loss of 22 to 9 neutrons. For the sake of brevity, the probability of formation is given for every other mass number only.

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Probability of Formation of Instantaneous Elements by Fusion
(Not corrected for boiling off of neutrons)

Instantaneous A	Instantaneous Z	Gaussian Factor	Gamow Factor	Relative Yield
202	79	.46	.17	.24
200	79	.61	.20	.38
198	78	.75	.23	.54
196	77	.88	.26	.72
194	76	.97	.28	.85
192	75	1.00	.31	.97
190	75	.97	.33	1.00
188	74	.88	.35	.96
186	73	.75	.37	.87
184	72	.61	.38	.72
182	71	.46	.40	.57
180	71	.32	.42	.42
178	70	.22	.43	.30
176	69	.13	.44	.18

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Probability of Formation of Chains of Mass A
(After consideration of the effect of boiling off of neutrons)

A	Relative Probability of Formation	Known Activity at End of Chain
190	.15	Unknown
188	.42	Unknown
186	.60	Unknown
184	.73	Unknown
182	.83	${}_{75}Re^{182}$ (13 h, 64 h) K, e^-
180	.94	Unknown
178	1.00	${}_{73}Ta^{178}$ (15.4 d) K, e^-
176	.97	${}_{73}Ta^{176}$ (8 h) K, e^-
174	.88	Unknown
172	.80	${}_{71}Lu^{172}$ (>100 d) K, e^-
170	.70	${}_{71}Lu^{170}$ (2.1 d) β^+
168	.58	Unknown
166	.47	${}_{69}Tm^{166}$ (7.7 h) β^+
164	.32	Unknown
162	.23	Unknown
160	.16	${}_{67}Ho^{160}$ (20 m)

Of these elements, two at least are positron emitters and so more easily detected. It is possible that there are others among the unknown isotopes which are positron active. The activity which can be collected is somewhat marginal. Consider for example the collected activity to be expected for the positron emitter ${}_{69}Tm^{166}$; assume 10^{-10} of the bomb is collected.

$$1.2 \times 10^{14} \times 10^{-10} \times .05 \times \frac{1}{7.7 \times 60} \approx 1 \text{ c/m}$$

The yield of fusion elements is proportional to

$$\frac{\sigma}{v_{rel}} \theta^3 \epsilon^2 \frac{V}{T}$$

where θ is the electron temperature, ϵ is the efficiency, and V is the reaction volume. In the case of the super both the volume and the electron temperature are expected to be larger than in an ordinary fission bomb. The super therefore offers an opportunity for an increased yield. Present plans are not well enough defined to justify a useful estimate of this increase, however.

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